Creep-Fatigue Analysis for LMFBR's Structures:
Identification of Chaboche's Models for the Stainless Steel 316 SPH

D. Rive, C. Escaravage, B. Riou
Framatome, Lyon, France

M. T. Cabrillat
Commissariat à l'Energie Atomique, Cadarache, France

ABSTRACT

The identification which is proposed concerns the Chaboche's plastic and
viscoplastic model as well as the Chaboche's continuum damage model.

This study will recall the constitutive equations of the considered models and
their possibilities linked to the required parameters.

Yet, emphasis will be put on methods carried out in the identification of
parameters. Moreover, the particular case of the stainless steel 316 SPH will
be treated and comparison with experimental results will be made.

1 INTRODUCTION

LMFBR's structures are highly solicited and numerical models to describe
precisely the behaviour of the materials are needed.

These models have however the drawback to require the knowledge of several
parameters that have to be determined with accuracy.

This study will deal with the identification of these parameters.

This identification is the first step of a research program concerning the
margins relative to creep-fatigue analysis applied in design of LMFBR's
structures.

2 THE CHABOCHE'S PLASTIC AND VISCOPLASTIC MODEL

In both plastic and viscoplastic models, the total strain may be written as :

\[ \varepsilon = \varepsilon^e + \varepsilon^p \]

where \( \varepsilon^e \) is the elastic strain and \( \varepsilon^p \) the inelastic strain (not dissociated from
the creep strain in viscoplasticity).

The equation of the viscoplastic model are recalled in table 2. It's to be
noticed that the whole behaviour of the material is linked to the calculation
of \( \varepsilon^p \) \( \left( \dot{\varepsilon}^p = \frac{\sigma}{\dot{\varepsilon}_0} \right) \).

3 PLASTIC AND VISCOPLASTIC IDENTIFICATION OF THE STAINLESS STEEL 316 SPH

3.1 Experimental data

Main experimental data come from the technical appendix A3.1s of the RCE-MR
which gives in particular :
- the young’s Modulus and Poisson’s ratio : E, \( \gamma \),
- the mean monotonous curves.


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The reduced cyclic curves used are deduced from the one recommended by the Design Construction and Rules Committee, valid from 20 °C to 600 °C:
\[ \frac{\Delta \sigma}{2} = A \left( \frac{\Delta \varepsilon}{2} \right)^n, \quad A = 440 \text{ MPa}, \quad n = 0.319. \]
Moreover, other experimental data are used including, hysteresis loops, cyclic consolidation curves and characteristics of the viscoplastic behaviour of the 316 SPH. The identification which is proposed is reliable for a total maximum strain of 1%.

3.2 Preliminary plastic identification:

This identification has been carried out as a preliminary task for each temperature 20 °C, 200 °C, 400 °C, 600 °C according to the following steps:

- \( \sigma_c \) can be chosen equate to 1 except when non negligible viscosity terms introduce the parameter \( \sigma_c \) which also weighted the isotropic variable \( R \).
- \( k, c_1, \gamma_1 \) and \( c_2 \) are obtained by comparing the monotonic curve with an equation derived from the model when \( R = 0, \gamma_1 = \gamma_2 \) and \( \gamma_1 > \gamma_2 \) (fig. 1).
- \( \Delta_2, \Delta_3 \) and \( C_4 \) are obtained as a function of \( \mu \) by comparing the cyclic curve to the equation derived from the model when \( \mu = 0.5 \).
- The values chosen for \( b \) and \( \mu \) were previously find adequate to modelize the cyclic consolidation behaviour of similar grades; in addition \( \gamma_2^* \) was fixed to 100 (\( \gamma_2^* < \gamma_1^* \)).

3.3 Viscoplastic identification

Two ways may be followed for this identification.

The first one consists in using a plastic identification, keeping in mind that plastic and viscoplastic calculations have to lead to the same results for a total strain rate of \( 10^3 \text{ s}^{-1} \). The second one consists in using the viscoplastic identification of a material with a behaviour close to the one studied.

In fact, a compromise of these two ways has been used. The identification has been undertaken according to the following steps:

* 1st step: identification at 600 °C
By neglecting the time-recovery terms, it is possible to express the evolution of the stress \( \sigma \) in tensile tests under the form:
\[ \sigma = k + (K_e + \alpha_d R)(\dot{\varepsilon})^{\text{in}} = \sigma + \alpha_d R \]
During tensile tests, \( R = 0 \) and the yield stress has to be compared with \( k + K_e (\dot{\varepsilon})^{\text{in}} \).
A compromise between creep tests and a reasonable value of \( k \) led to \( k = 35 \text{ MPa} \) and \( K_e = 70 \text{ MPa} \).

The cyclic curves are well represented with \( Q_a = 40 \text{ MPa} \) and \( Q_m = 460 \text{ MPa} \). The other parameters are the same than those given by J.L. Chaboche (Chaboche et al. [1987]).

It's interesting to note that:

- \( \eta = 0.04 \) induces a non-instantaneous memorization.
- The value of \( C_4 \) is lower than the one of plastic case because \( k + K_e (\dot{\varepsilon})^{\text{in}} \), the initial value on monotonic curve, is higher in the viscoplastic case.

* 2nd step: identification at 400 °C
At this temperature, the sensitivity to strain rate and time-recovery effect are no more experimentally detectable.
In consequence, the identification is carried out on the base of the one at 600 °C where the viscosity terms have been saturated and the recovery terms have been put to zero.
It's however to be noticed that it is not possible for numerical reasons to put $K$ to zero or to make $n$ tend to an infinite value. Then, saturation of viscosity is obtained with $K = 10$ and $\alpha_k = 0$. Moreover, $k$ has to be increased in compatibility with the monotonic curve. Finally, as $\alpha_k$ has been put to zero, $\alpha_n$ may take the same value than the one of the plastic case $\alpha_k = 1$ and $Q_n$ and $Q_w$ have to be modified in agreement with the cyclic curve (fig. 1).

* 3rd step : identification at 200 °C and 20 °C
On the basis of the 400 °C identification, $k$ has been increased from 95 MPa to 135 MPa (200 °C) then 200 MPa (20°) so as to take into account the hardening of the material when the temperature decreases.
Moreover, $Q_n$ and $Q_w$ have been modified in agreement with this. The viscoplastic identification is proposed in table 3.
The figure 2 shows the comparison between the viscoplastic calculation, the RCC-MR and the experiments on cyclic consolidation curves.

4 THE CHABOCHÉ’S CONTINUUM DAMAGE MODEL

The Chaboche's model corresponds to a mathematical approach of the continuum damage mechanics.
In this approach, the damage of a volume element is characterized by a damage variable $D$, $0 \leq D \leq 1$, where $D = 0$ corresponds to the undamaged state and $D = 1$ to the rupture of the element.
The total damage in creep-fatigue interaction is defined by:
$$dD = dD_{\text{creep}} + dD_{\text{fatigue}}$$

J.L. Chaboche has proposed the following differential relations:

- for Creep : $dD_{\text{creep}} < \frac{\chi(g)}{\lambda} \gamma (1 - D)^{\lambda}$
with $<\gamma>$ is the positive part of $\gamma$
$\chi(g)$ is a creep rupture criterion depending on the stress tensor.

- for fatigue : $dD_{\text{fatigue}} = [1 - (1 - D)^{2 + 1}] = A(\Delta \sigma , \bar{\sigma}_n)^B$
with $\Delta = 1 - \sigma B(\Delta \sigma , \bar{\sigma}_n)$ and $\bar{\sigma}_n = \frac{1}{3} tr \left( \frac{\sigma_{\text{max}} + \sigma_{\text{min}}}{2} \right)$

$A$ and $B$ are functions of the stress amplitude $\Delta \sigma$ and of the mean stresses parameter $\bar{\sigma}_n$. They are also dependent on the two material parameters to be identified ($b, M(o)$) as well as the instantaneous failure stress $\sigma_\alpha$.

5 CHABOCHÉ’S DAMAGE MODEL IDENTIFICATION IN THE CASE OF THE STAINLESS STEEL 316 SPH.

5.1 Fatigue parameters

The fatigue endurance may be written as a function of the mean stress and compared with the Goodman equation : this leads to $b = \frac{1}{\bar{\sigma}_n}$
The fatigue endurance curve for uniaxial cyclic tests may be written as:
$$N_f = N(0, 1) = \frac{1}{(1 - a)(\beta + 1)} \left[ \frac{\sigma_{\text{max}} - \bar{\sigma}}{M(o)(1 - D\sigma)} \right]^B$$
\( \sigma_u \) being the maximum stress and \( \bar{\sigma} \) the mean stress during cycling. With 
a = 0.2 at 600 °C and 0.9 at 200 °C with linear interpolation at intermediate 
temperatures, it is possible to derive \( \beta \) and \( M(0) \) from the fatigue endurance 
data.

5.2 Creep parameters

For this identification, through a lack of experimental data, we consider that 
\( f(s) = J(s) \). This means that tensile and compressive states won’t be 
distinguished.

In consequence, there are 3 parameters to identify: \( A, \alpha \) and \( k \).

For uniaxial tests, the time to failure under pure creep may be modeled by 
the relation:

\[
t_c = t(0, 1) = \frac{1}{k + 1} \left( \frac{|s|}{\Delta} \right)^r
\]

Once compared with experimental results, it leads to \( r \) and \( \Delta \) as a function 
of \( k \).

\( k \) will be given by creep-fatigue tests. In our case, \( k \) has been chosen to 5 
for all temperatures.

5.3 Creep-fatigue interaction

No other parameter than those formerly described needs to be introduced.

All the parameters of Chaboche’s damage model are presented in table 1.

The figure 3 shows the agreement between this model and experiments on creep 
tests.

6 CONCLUSION

This study has presented the identification of: Chaboche’s behaviour and 
damage model and has shown the good agreement with uniaxial tests.

These models have now to be validated on multiaxial tests and this is the 
second part of the research program where a mock-up representative of MAFER’s 
structures will be analyzed.

REFERENCE :

17-12 SPH sous chargements cycliques de longue durée. Document GIS rupture à 
chaud n° 11, p. 17.

| TABLE 1 : IDENTIFICATION OF CHAHOCE’S CONTINUUM DAMAGE MODEL FOR 
THE STAINLESS STEEL 316 SPH |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
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<tbody>
<tr>
<td>Material parameters</td>
<td>20 °C</td>
<td>450 °C</td>
<td>550 °C</td>
<td>600 °C</td>
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<tr>
<td>( \beta ) (MPa(^{-1}))</td>
<td>4.456</td>
<td>4.368</td>
<td>4.397</td>
<td>4.438</td>
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<td>( b ) (MPa)</td>
<td>0.00111</td>
<td>0.00167</td>
<td>0.00167</td>
<td>0.00167</td>
</tr>
<tr>
<td>( M(0) ) (MPa)</td>
<td>2781 a ( 0.28 )</td>
<td>2643 a ( 0.25 )</td>
<td>2448 a ( 0.22 )</td>
<td>2093 a ( 0.20 )</td>
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<tr>
<td>( \sigma_r ) (MPa)</td>
<td>200</td>
<td>180</td>
<td>170</td>
<td>150</td>
</tr>
<tr>
<td>( \sigma_s ) (MPa)</td>
<td>900</td>
<td>600</td>
<td>600</td>
<td>600</td>
</tr>
<tr>
<td>( \Lambda ) (MPa.s(^{0.5}))</td>
<td>( 8053(k+1)^{0.89} )</td>
<td>( 3725(k+1)^{0.16} )</td>
<td>( 2925(k+1)^{0.34} )</td>
<td>( 2467(k+1)^{1.16} )</td>
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<td>( \gamma )</td>
<td>14.481</td>
<td>8.805</td>
<td>7.485</td>
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<td>( \delta )</td>
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<td>0</td>
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<td>( \kappa )</td>
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<td>5</td>
<td>5</td>
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<td>( a )</td>
<td>0.9</td>
<td>0.381</td>
<td>0.260</td>
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TABLE 2: EQUATIONS OF THE VISCOPLASTIC MODEL

**Kinematic hardening**

\[ X = X_1 + X_2 \]

\[ (k_{i})_{N\text{med}} = \frac{2}{3} Cl \varepsilon^p - \gamma_i X_i \rho \]

\[ \gamma_i = \gamma_i (I_6 - (1 - a_i) \sigma^p - \eta) \]

**Isotropic hardening**

\[ \gamma = \gamma (I_6 - (1 - a_i) \sigma^p - \eta) \]

**Plastic strain nomenclature**

\[ \sigma = \sigma_0 + \sigma_0 (1 - e^p - \eta) \]

\[ P = \frac{3}{2} J(s^p - 1) - \sigma \cdot 0 \]

**Viscosity**

\[ \eta = \eta \left( \frac{\sigma - \sigma_0}{\sigma_0} \right) \]

**Viscoplastic potential**

\[ \sigma - \frac{1}{n + 1} \left( \frac{\dot{\varepsilon}}{\dot{\varepsilon}} \right)^{n+1} \]

\[ \sigma = \sigma_0 (\sigma^p - 1) \]

\[ \sigma_{\text{op}} = \sigma_{\text{op}} (\sigma^p - 1) \]

\[ \sigma_{\text{op}} = \sigma_{\text{op}} (\sigma^p - 1) \]

**Time-recovery for kinematic hardening**

\[ \dot{\gamma} = (\dot{\gamma})_{\text{med}} - \gamma_0 \dot{\varepsilon}^p (\dot{\varepsilon})^{p-1} \]

**Time-recovery for isotropic hardening**

\[ \dot{\gamma} = (\dot{\gamma})_{\text{med}} \]

**TABLE 3: VISCOPLASTIC IDENTIFICATION OF THE STAINLESS STEEL 316 STH**

<table>
<thead>
<tr>
<th>Material Parameters</th>
<th>20 °C</th>
<th>200 °C</th>
<th>400 °C</th>
<th>600 °C</th>
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<td>ν</td>
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<td>k (MPa)</td>
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<td>γi (MPa)</td>
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<tr>
<td>c (MPa)</td>
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<td>1 950</td>
<td>1 950</td>
<td>1 950</td>
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<tr>
<td>γi (2)</td>
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</table>
FIGURE 1

Monotonic curve

Viscoelastic Identification:
Significance of parameters

$\gamma_1$, $\gamma_2$

Tests of different strain rates
$\gamma$, $K_e$, $n$

Cyclic consolidation curve

$\overline{V_{\text{max}}}$

$A$, $b$, $\eta$

Cyclic curve

$\Delta E$ $\overline{c}$

$C_2=2\mu(Q_m-Q_e)$

$\overline{\gamma}$, $Q_{\text{m}}$, $Q_{\text{e}}$

Creep relaxation curves

$\overline{\theta}$, $\overline{m}$, $\overline{\gamma_1}$, $\overline{m_1}$, $\overline{\gamma_2}$, $\overline{m_2}$

FIGURE 2

Simulation of cyclic consolidation curves
in fatigue-relaxation after viscoelastic identification

Fatigue-relaxation (I = 10 minutes in tension
$\eta = 0.35$, $A = 10^{-3}$)

FIGURE 3

Time to failure under creep in the case
of the stainless steel 316 S/N